NiAS セミナー

長崎総合科学大学　出前講義

数学で語る船の波

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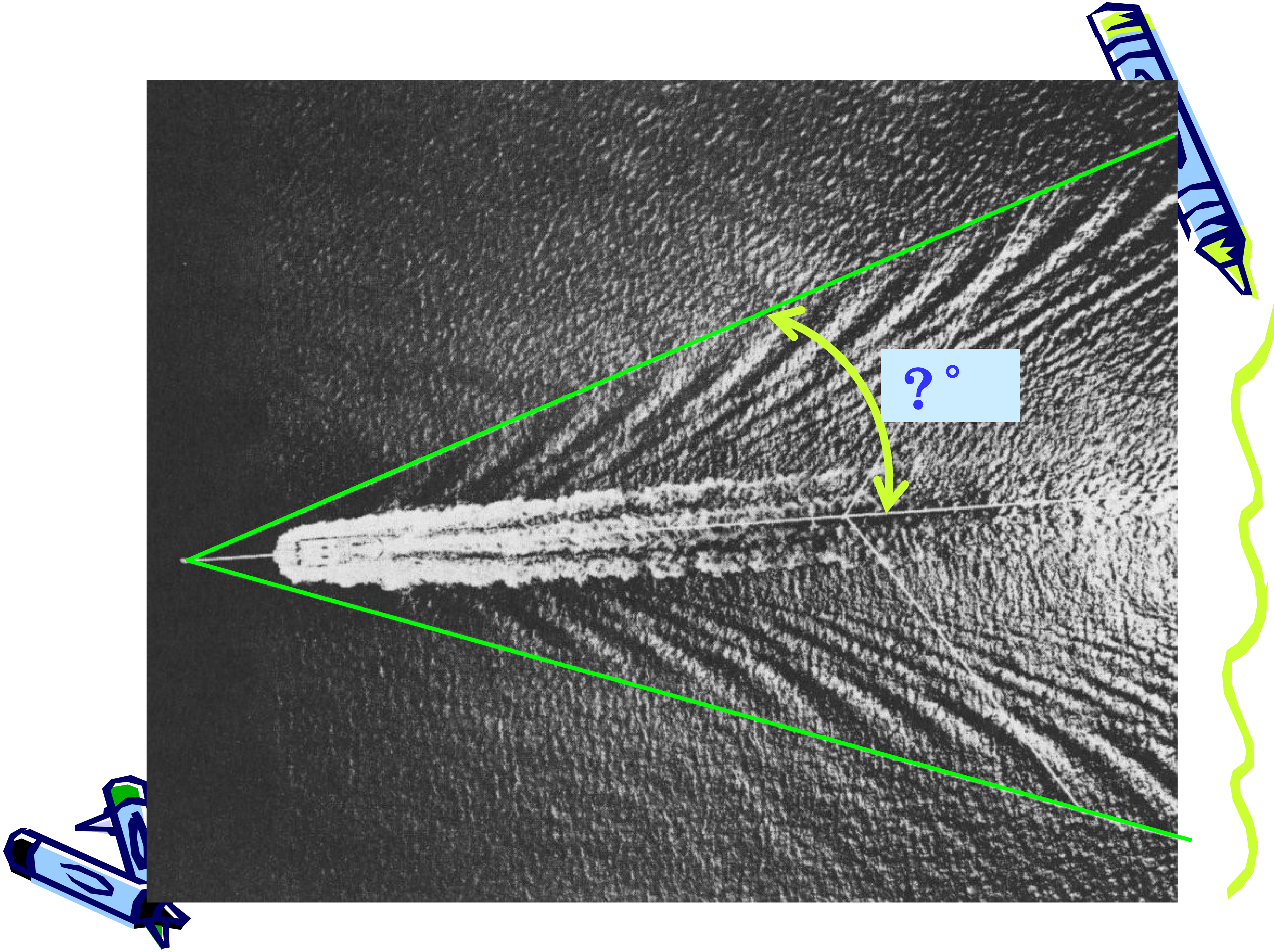
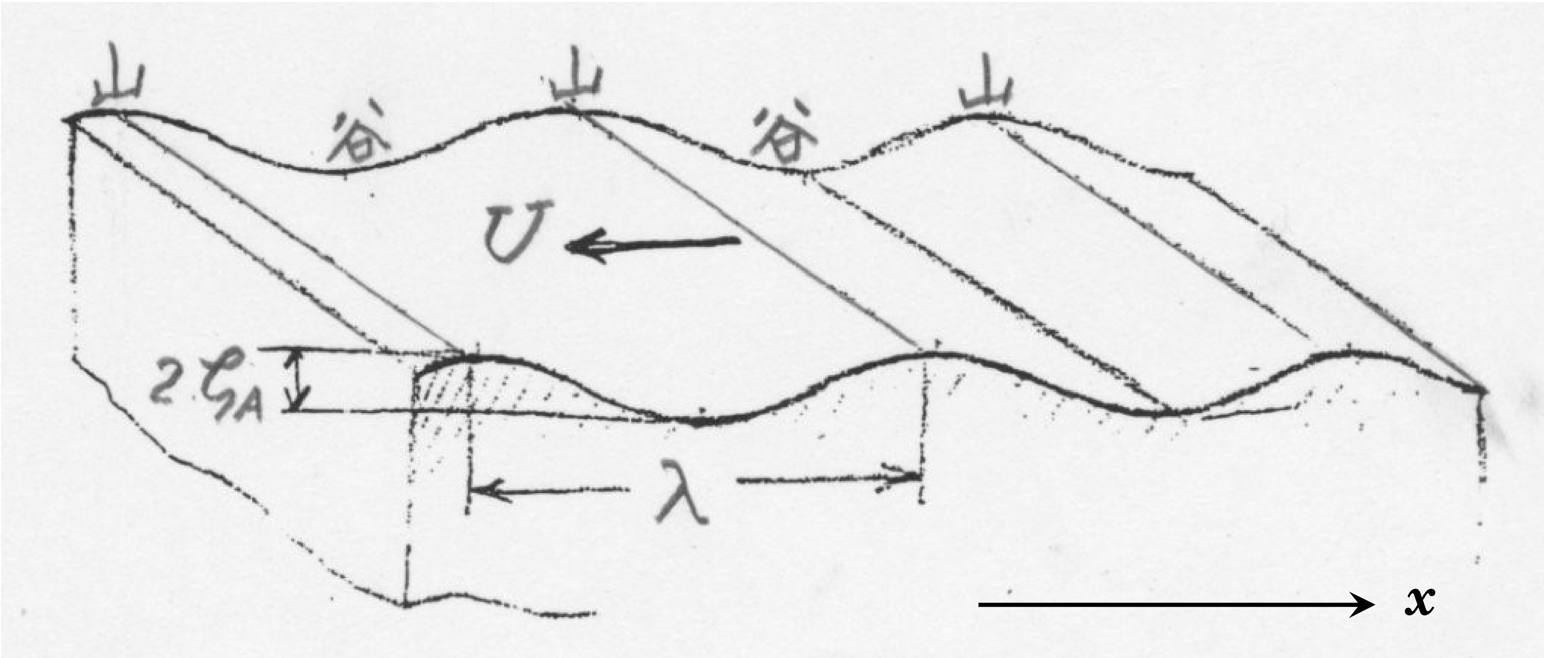


2次元の波（平面波）

*x*の負軸方向に波速*U*で進行する

$$\zeta = \zeta_A \cos(kx + \omega t)$$



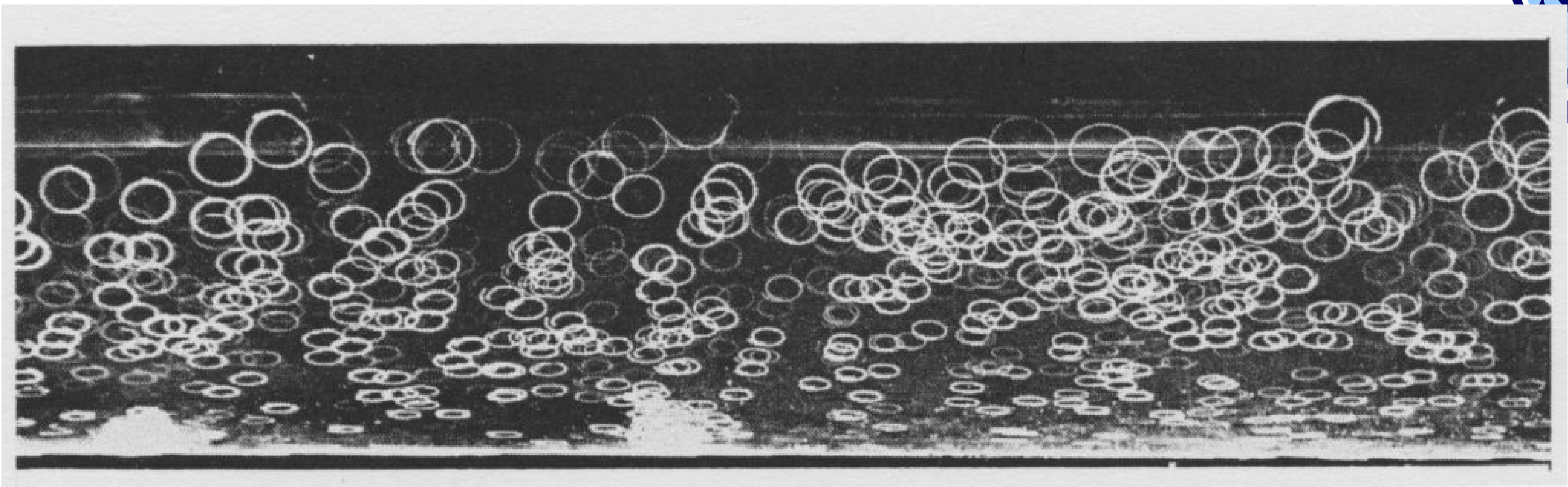
$$\lambda = UT$$

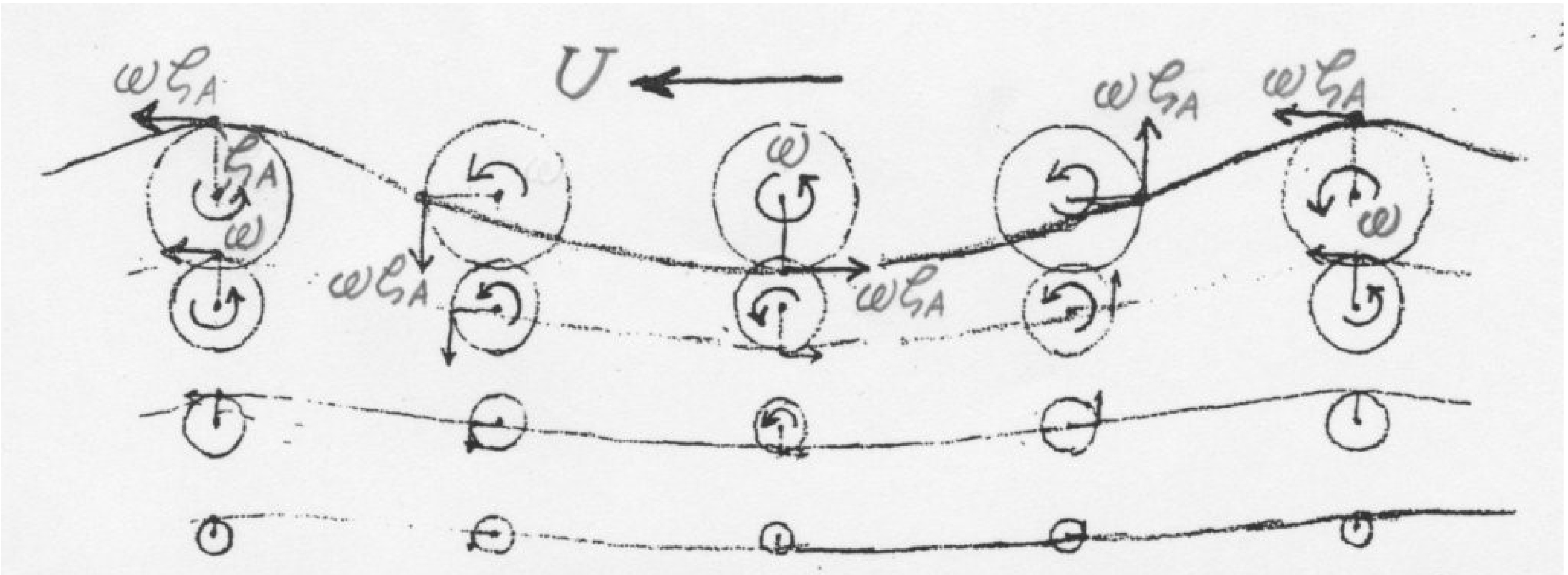
周波数ωと，波数kの関係

$$\frac{2\pi}{k} = U\frac{2\pi}{\omega}$$

$$\omega = kU \quad \cdots\cdots(a)$$

進行波中の水粒子のOrbital Motion



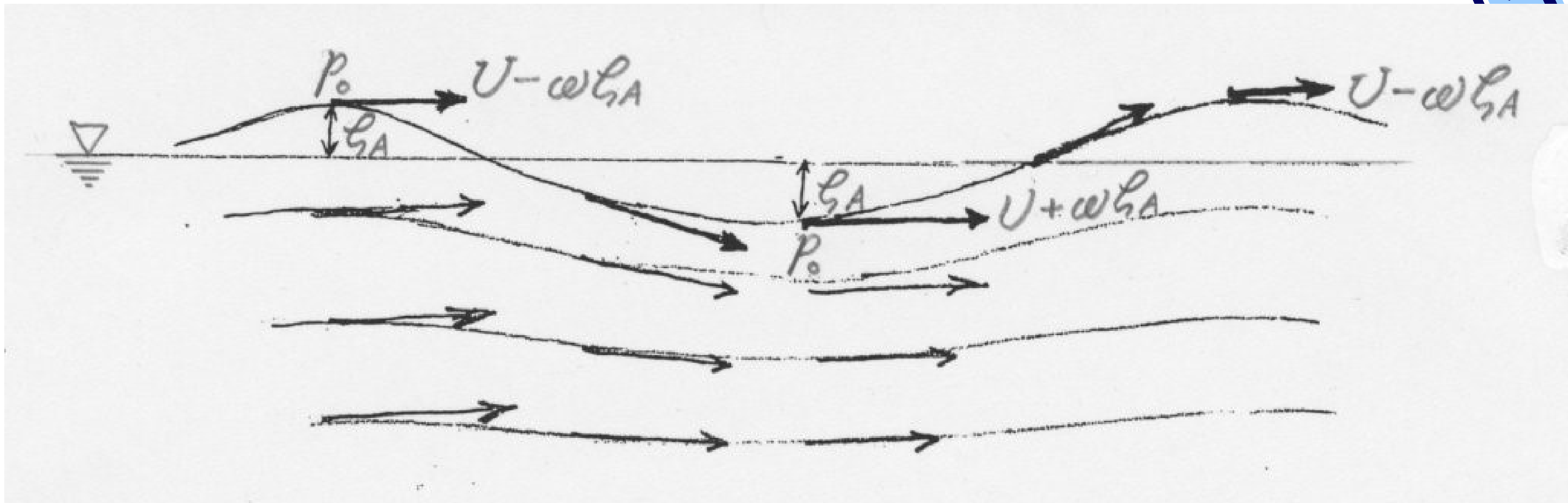


水粒子の回転速度＝$\omega\zeta_A$

$$\omega = \frac{2\pi}{T}$$

この系全体に，U（波の進行速度と逆向き）を付け加える．

波は空間に停止する．→ 定常問題　$\zeta = \zeta_A \cos(kx + \varepsilon)$



Bernoulli の定理（圧力＋運動＋位置の３ｴﾈﾙｷﾞｰが，保存される.）

山 →　$$\frac{p_0}{\rho} + \frac{1}{2}(U - \omega\zeta_A)^2 + g\zeta_A = \frac{p_0}{\rho} + \frac{1}{2}(U + \omega\zeta_A)^2 - g\zeta_A$$　← 谷

$$-\omega\zeta_A U + g\zeta_A = \omega\zeta_A U - g\zeta_A$$

$$2\omega\zeta_A U = 2g\zeta_A$$

周波数ωと，重力加速度gの関係

$$\omega = \frac{g}{U} \quad \cdots\cdots(b)$$

水波（重力波）の分散関係

$$\omega = kU \quad \cdots\cdots(a) \qquad \omega = \frac{g}{U} \quad \cdots\cdots(b)$$

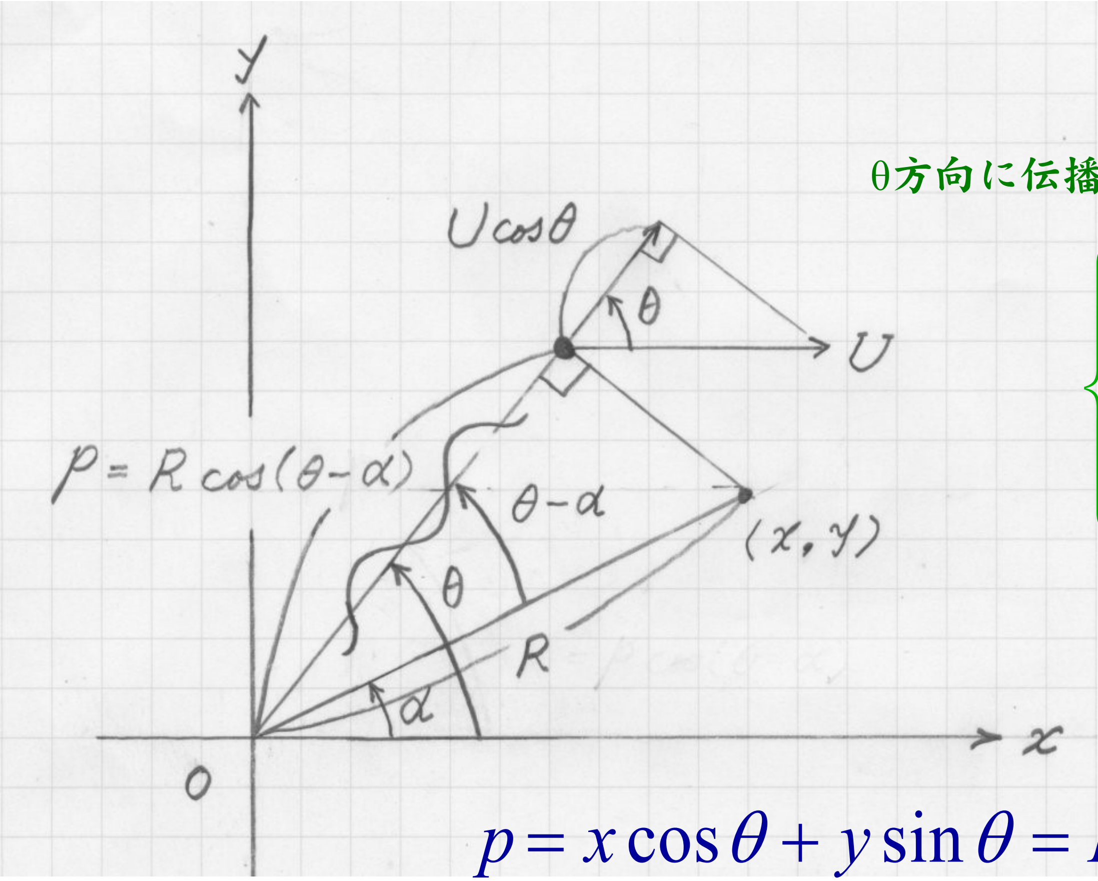
$$kU = \frac{g}{U}$$

波数$k\left(=\dfrac{2\pi}{\lambda}\right)$

$$k = \frac{g}{U^2}$$

$$\lambda = \frac{2\pi U^2}{g}$$

素成波（2次元波）の重畳による船の波（3次元波）



θ方向に伝播する素成波の進行方向の

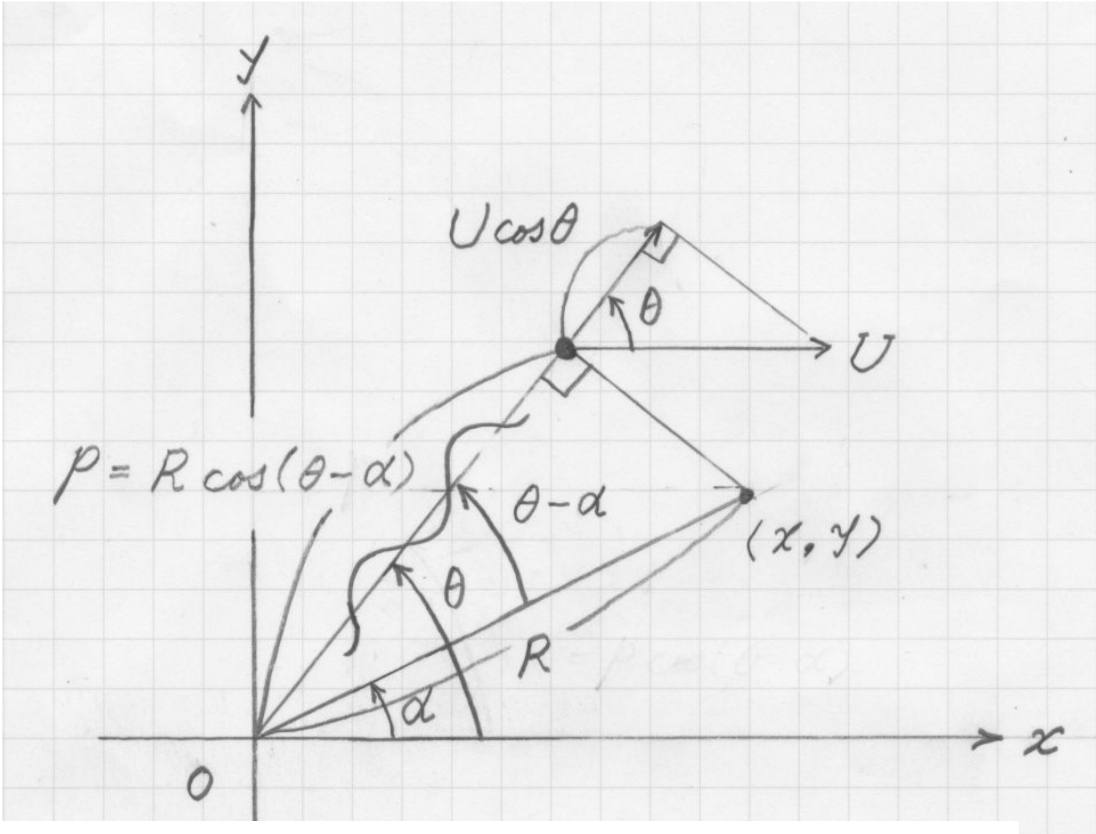
- 座標 p
- 波速 $U\cos\theta$
- 波数 $k(\theta)$

$$p = x\cos\theta + y\sin\theta = R\cos(\theta-\alpha)$$

$$k(\theta) = \frac{g}{(U\cos\theta)^2} = \frac{g}{U^2}\sec^2\theta = K_0\sec^2\theta$$

$$\lambda(\theta) = \frac{2\pi}{k(\theta)} = \frac{2\pi}{K_0}\cos^2\theta = \lambda_0\cos^2\theta$$

素成波（2次元波）の重畳による船の波（3次元波）

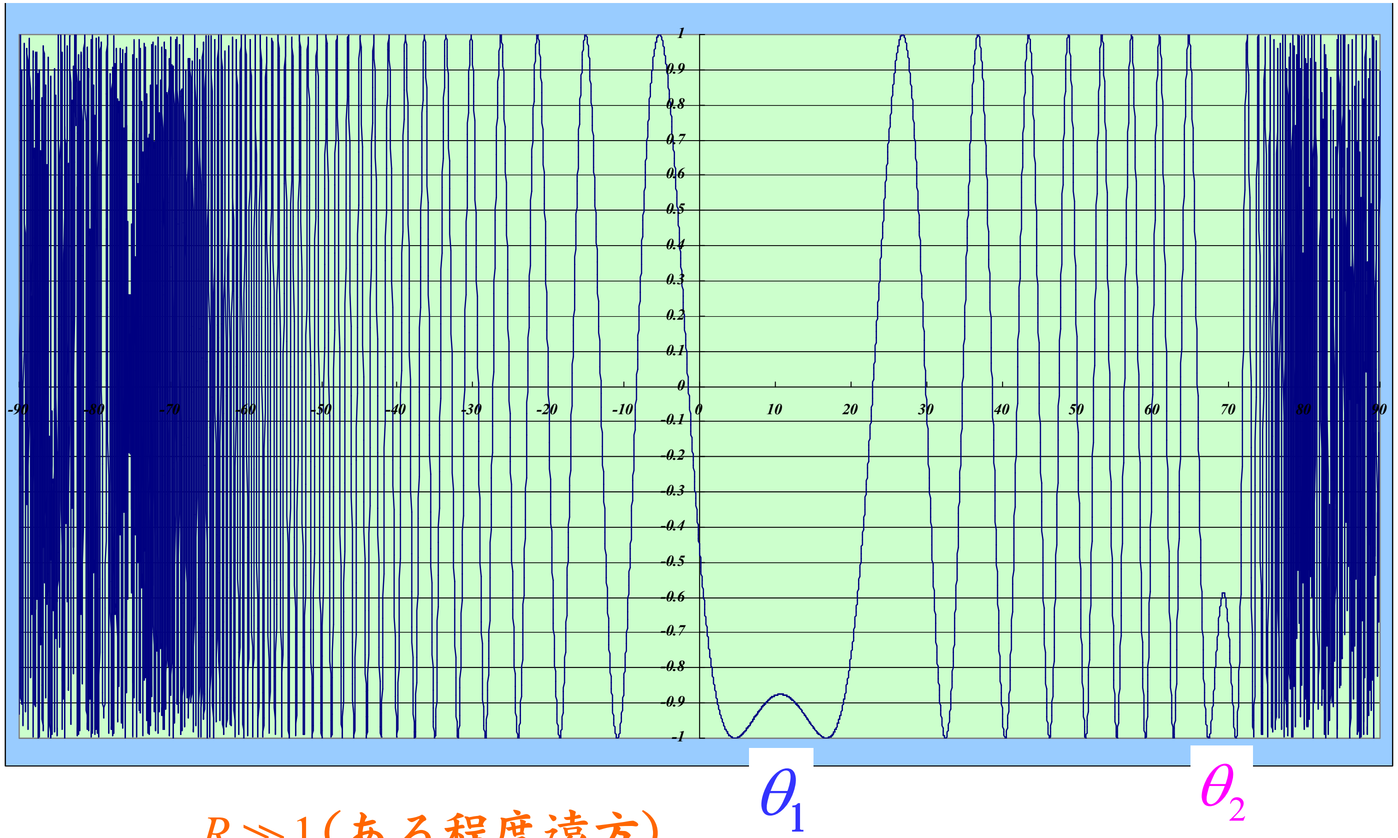


$$\zeta(x,y)=\sum_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}}\zeta_A(\theta)\cos(k(\theta)p(\theta)+\varepsilon(\theta))\delta\theta$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{C(\theta)\cos(K_0 p\sec^2\theta)+S(\theta)\sin(K_0 p\sec^2\theta)\right\}d\theta$$

停留位相法による波高ζの算定

$$\zeta = \left(\int_{-\frac{\pi}{2}}^{\theta_1-\varepsilon} + \int_{\theta_1-\varepsilon}^{\theta_1+\varepsilon} + \int_{\theta_2-\varepsilon}^{\theta_1+\varepsilon} + \int_{\theta_2-\varepsilon}^{\theta_2+\varepsilon} + \int_{\theta_2+\varepsilon}^{\frac{\pi}{2}}\right) C(\theta)\cos(K_0 \overbrace{\boxed{R\cos(\theta-\alpha)}}^{p} \sec^2\theta)\,d\theta$$



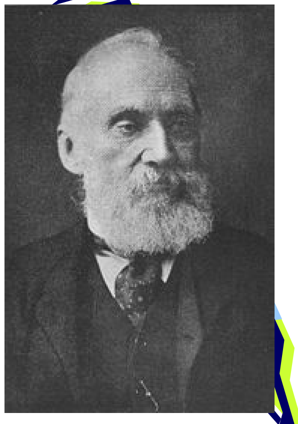
$R \gg 1$（ある程度遠方）

$$\zeta \fallingdotseq \left(\int_{\theta_1-\varepsilon}^{\theta_1+\varepsilon} + \int_{\theta_2-\varepsilon}^{\theta_2+\varepsilon}\right) C(\theta)\cos(K_0 R\cos(\theta-\alpha)\sec^2\theta)\,d\theta$$

停留位相法による解析

ケルヴィン卿 (Lord Kelvin) 1887

本名：*William Thomson* (1824〜1907)



$$\zeta = \left(\int_{\theta_1-\varepsilon}^{\theta_1+\varepsilon} + \int_{\theta_2-\varepsilon}^{\theta_2+\varepsilon}\right) C(\theta)\cos(K_0 R\, F(\theta))\, d\theta$$

$F'(\theta_1) = 0$　　$F'(\theta_2) = 0$　　但し，$F(\theta) = \cos(\theta - \alpha)\sec^2\theta$

$$F'(\theta) = \frac{dF}{d\theta} = \{2\tan\theta - \tan(\theta-\alpha)\}\cos(\theta-\alpha)\sec^2\theta = 0$$

$$2\tan\theta - \tan(\theta-\alpha) = 0$$

$$2\tan\theta - \frac{\tan\theta - \tan\alpha}{1+\tan\theta\tan\alpha} = 0 \quad \leftarrow \text{加法定理}$$

$$2\tan^2\theta\tan\alpha - \tan\theta + \tan\alpha = 0$$

$$\tan\theta = -\frac{1 \mp \sqrt{1 - 8\tan^2\alpha}}{4\tan\alpha} \qquad \tan\alpha = -\frac{\tan\theta}{1+2\tan^2\theta}$$

*Kelvin*波の存在範囲

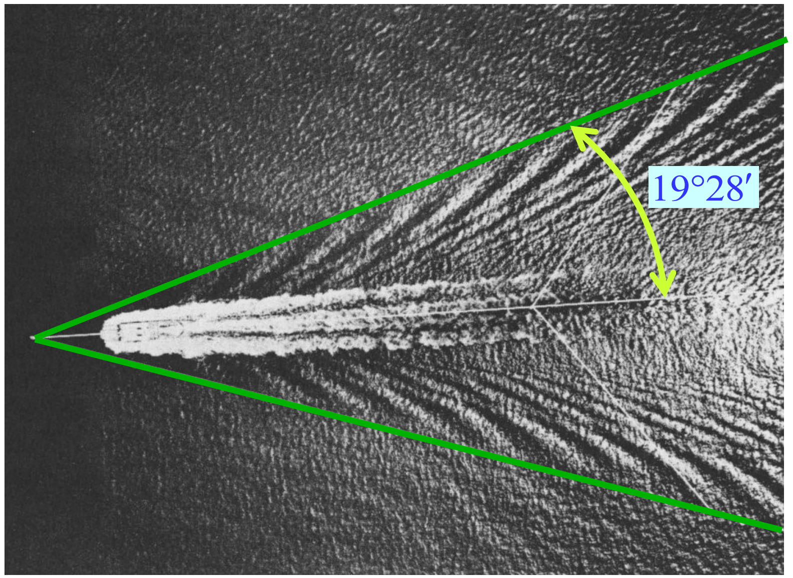
$$\tan\theta_{\substack{1\\2}} = -\frac{1 \mp \sqrt{1-8\tan^2\alpha}}{4\tan\alpha}$$

$$1-8\tan^2\alpha \geq 0$$

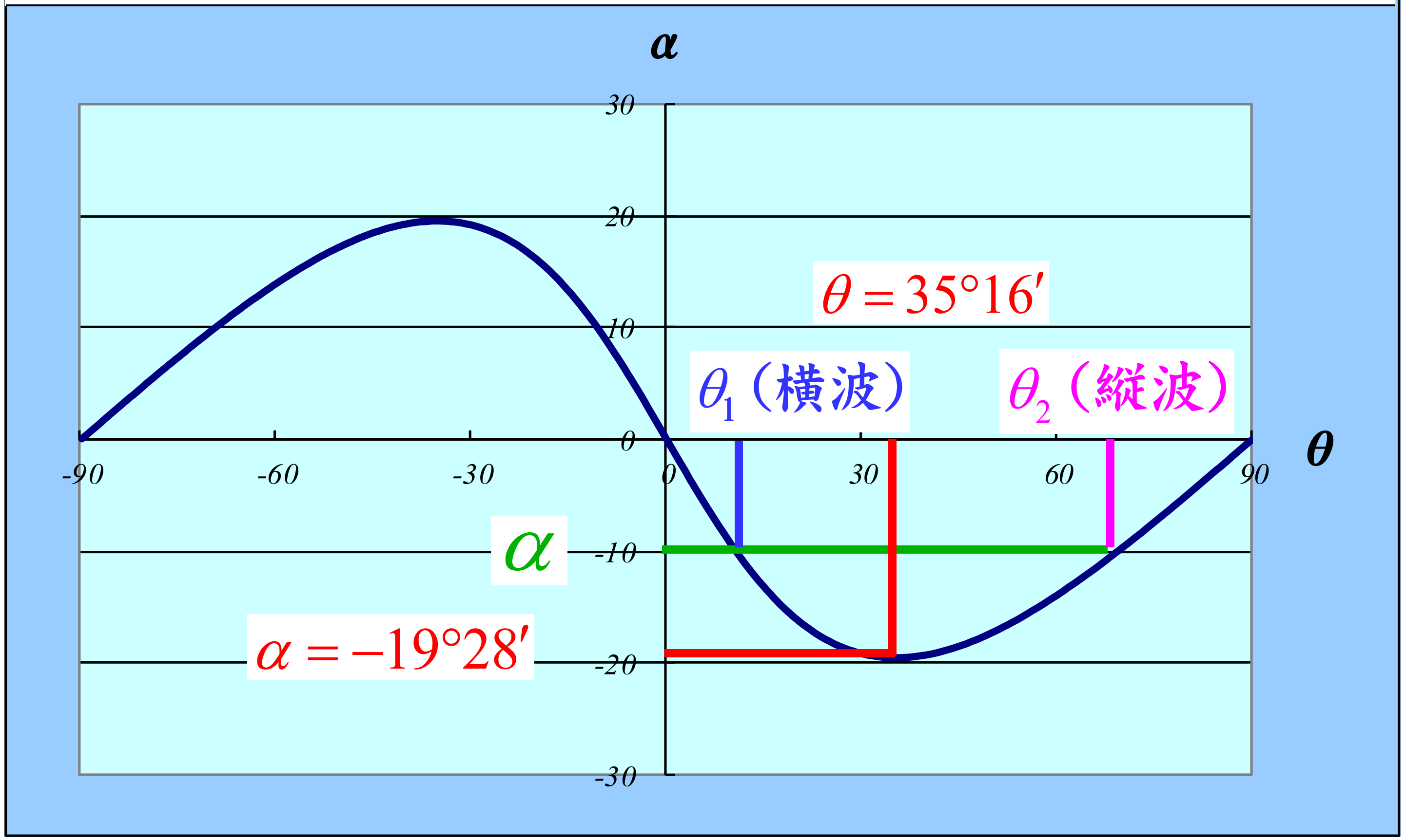
$$\tan^2\alpha \leq \frac{1}{8}$$

$$-\frac{1}{\sqrt{8}} \leq \tan\alpha \leq \frac{1}{\sqrt{8}}$$

$$-19°28' \leq \alpha \leq 19°28'$$



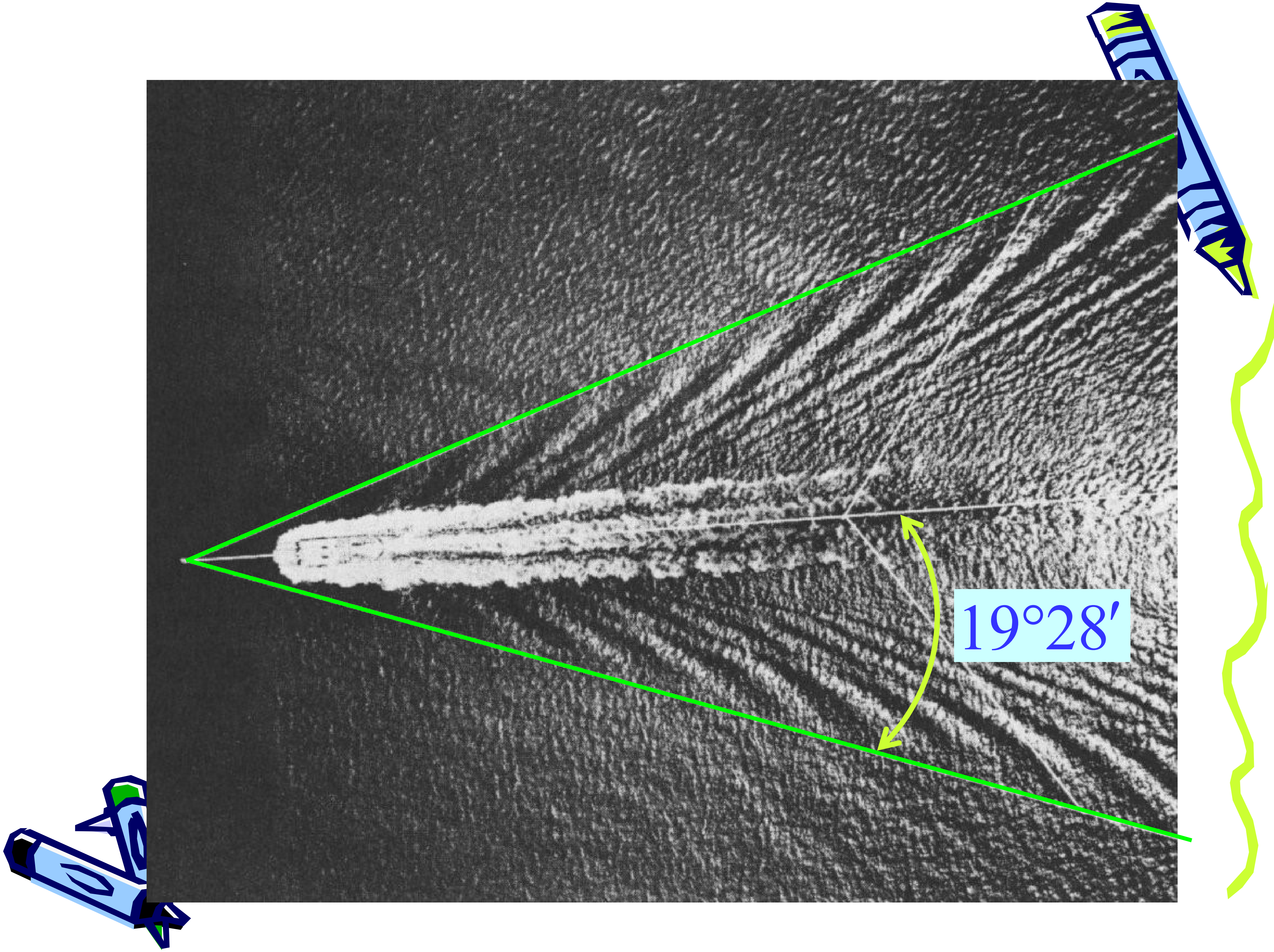
α （波高ζの位置）と θ （素成波の伝播方向）



$$\tan\alpha = -\frac{1}{\sqrt{8}}$$

$$\tan\theta = -\frac{1 \mp \sqrt{1-8\cdot\frac{1}{8}}}{4\cdot\left(-\frac{1}{\sqrt{8}}\right)} = \frac{\sqrt{8}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\begin{cases} \alpha = \tan^{-1}\left(-\frac{1}{\sqrt{8}}\right) = -19.47° = -19°28' \\ \theta = \tan^{-1}\frac{1}{\sqrt{2}} = 35.26° = 35°16' \end{cases}$$


19°28′

波頂線（等位相線）の決め方

$$\zeta = \left(\int_{\theta_1-\varepsilon}^{\theta_1+\varepsilon} + \int_{\theta_2-\varepsilon}^{\theta_2+\varepsilon} \right) C(\theta) \cos(K_0 p \sec^2 \theta)\, d\theta$$

等位相の条件（素成波の山谷）

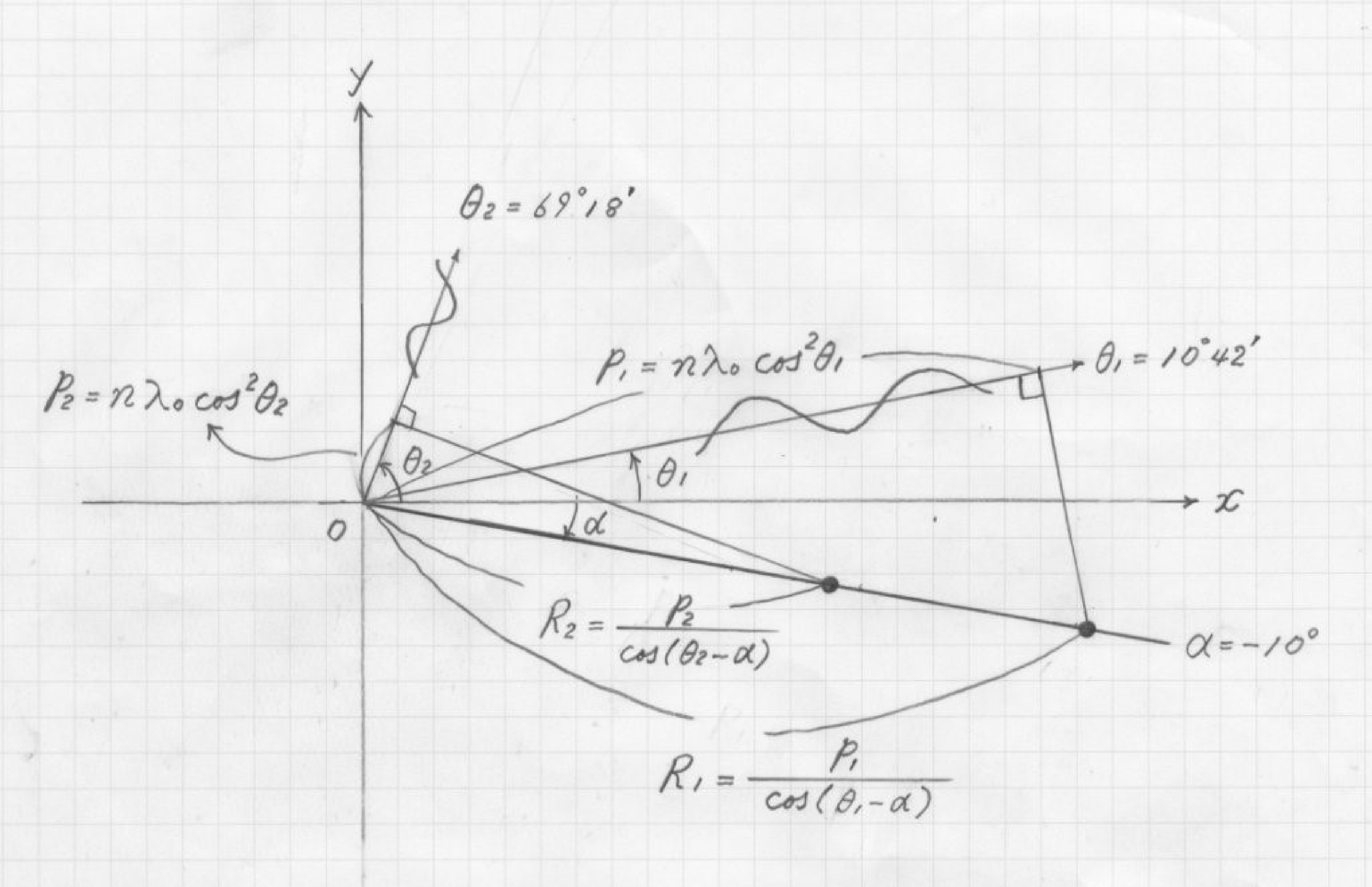
$$K_0 p \sec^2 \theta = 2\pi n \;(= Const.)$$

但し，$n = 0, 1, 2, 3, \cdots\cdots$

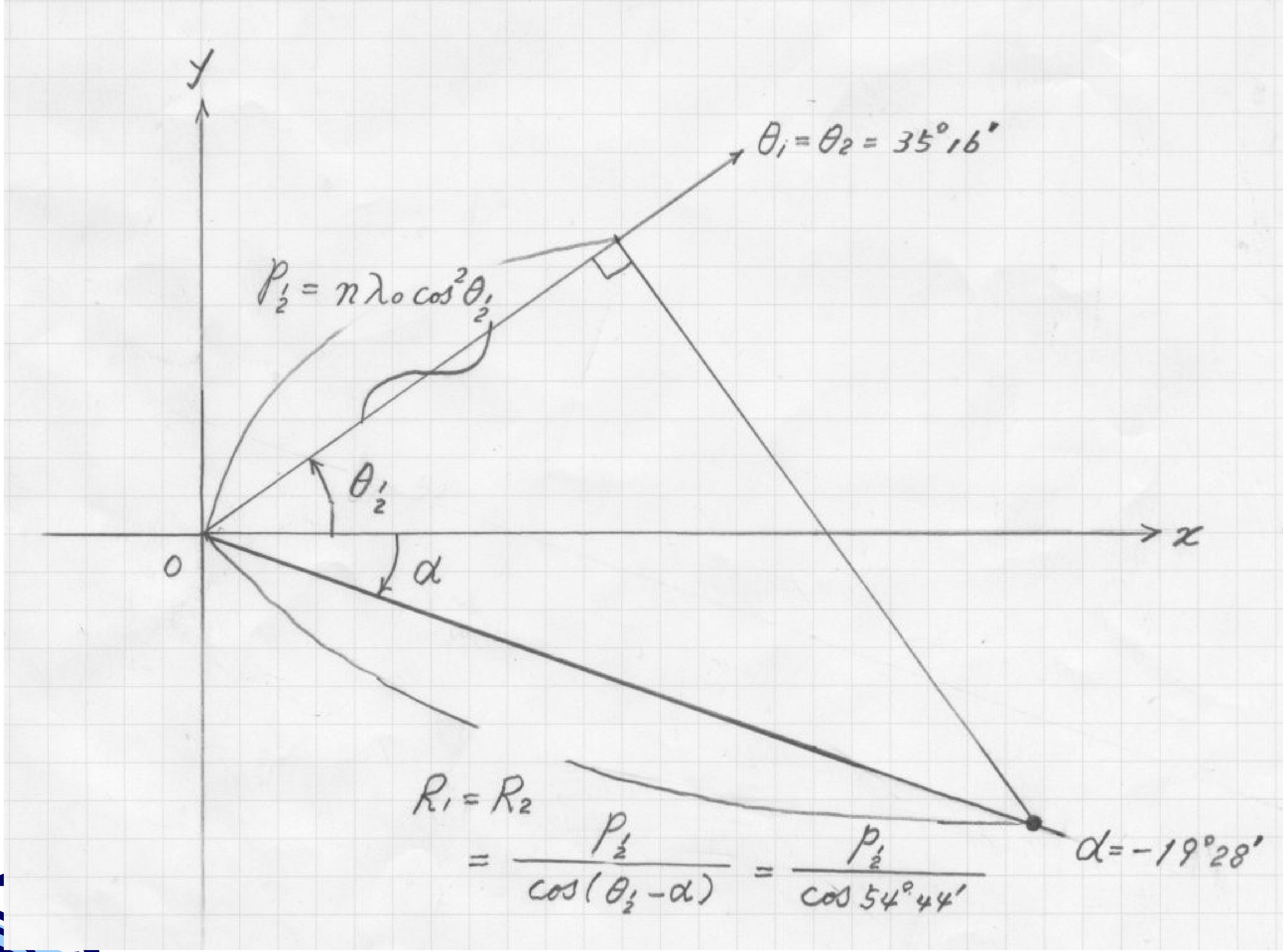
$$p(\theta) = \frac{2\pi n}{K_0} \cos^2 \theta = n \lambda_0 \cos^2 \theta$$

（$=\theta$方向に進む素成波の波長$\lambda_0 \cos^2 \theta$の整数n倍）

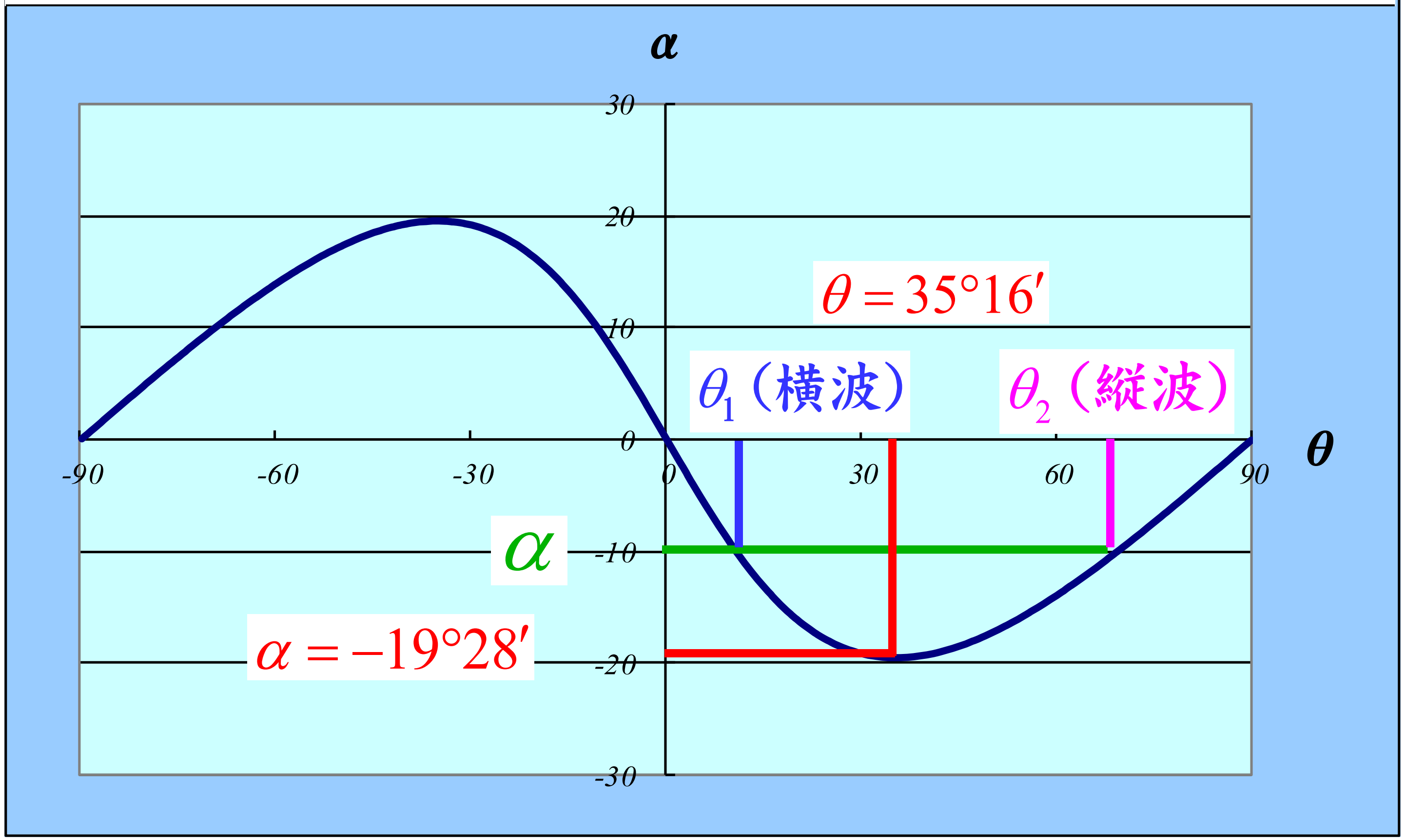
$$p(\theta) \propto \cos^2 \theta$$



y
x
0
θ2 = 69°18'
θ1 = 10°42'
P2 = nλ0 cos²θ2
P1 = nλ0 cos²θ1
θ2
θ1
α
R2 = P2 / cos(θ2 − α)
R1 = P1 / cos(θ1 − α)
α = −10°


y
θ1 = θ2 = 35°16'
P1/2 = nλo cos² θ1/2
θ'2
0
x
α
R1 = R2
= P1/2 / cos(θ1/2 − α) = P1/2 / cos 54°44'
α = −19°28'

α（波高ζの位置）と θ（素成波の伝播方向）



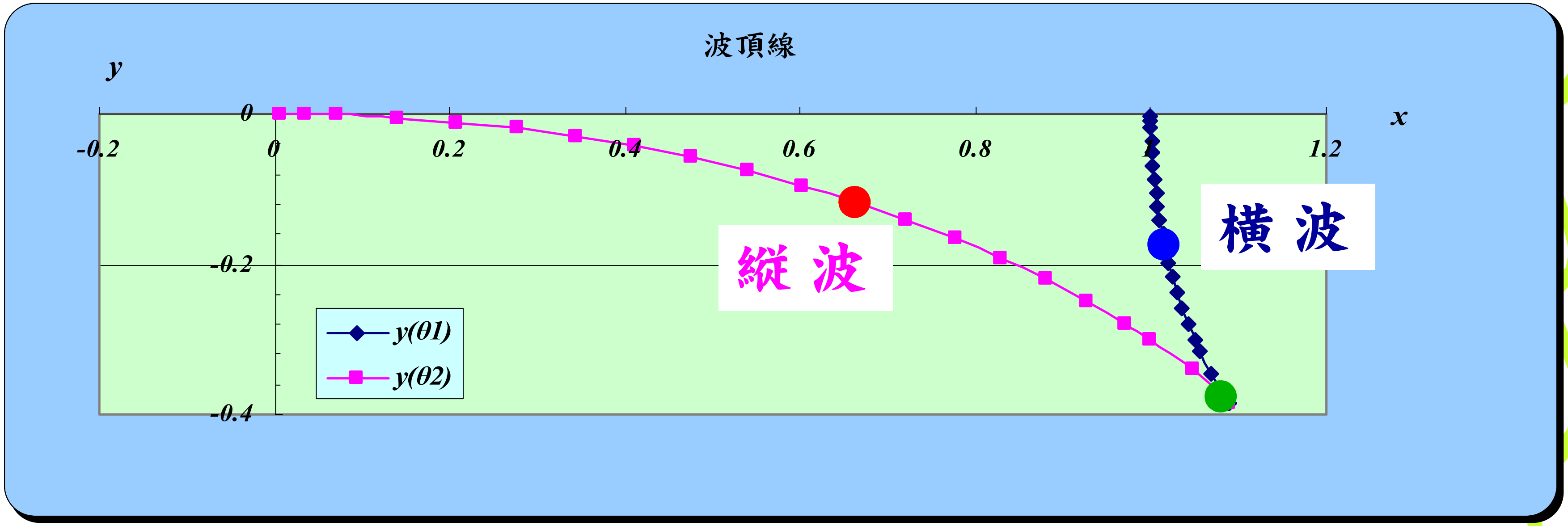
$$\tan\alpha = -\frac{1}{\sqrt{8}}$$

$$\tan\theta = -\frac{1 \mp \sqrt{1 - 8\cdot\frac{1}{8}}}{4\cdot\left(-\frac{1}{\sqrt{8}}\right)} = \frac{\sqrt{8}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

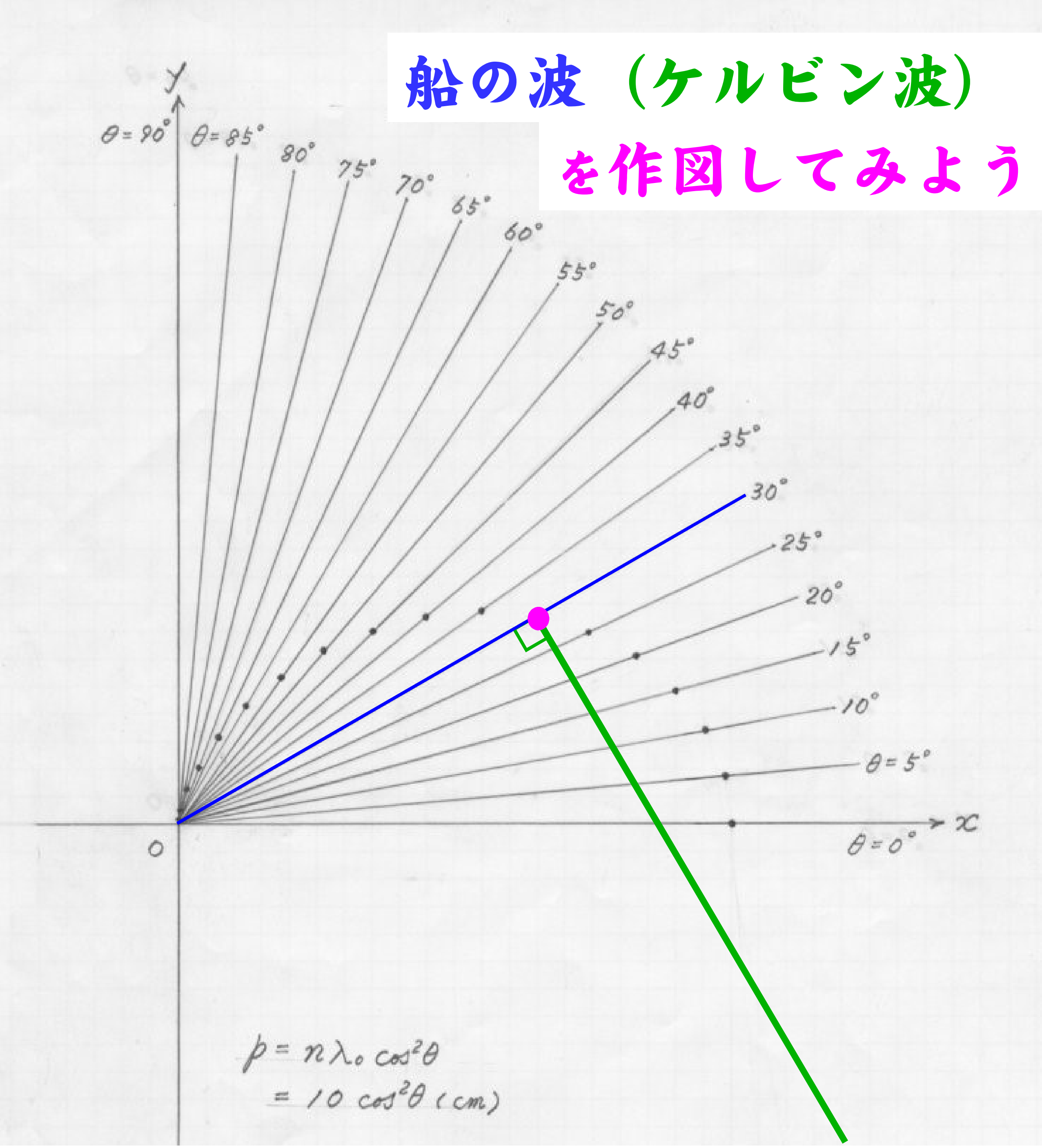
$$\begin{cases} \alpha = \tan^{-1}\left(-\frac{1}{\sqrt{8}}\right) = -19.47^\circ = -19^\circ 28' \\ \theta = \tan^{-1}\frac{1}{\sqrt{2}} = 35.26^\circ = 35^\circ 16' \end{cases}$$

α	θ_1	θ_2	$p(\theta_1)$	$p(\theta_2)$	$R(\theta_1)$	$R(\theta_2)$	$x(\theta_1)$	$y(\theta_1)$	$x(\theta_2)$	$y(\theta_2)$
-0.10	0.1	89.8	1.000	0.000	1.000	0.007	1.000	-0.002	0.007	0.000
-0.50	0.5	89.0	1.000	0.000	1.000	0.035	1.000	-0.009	0.035	0.000
-1.00	1.0	88.0	1.000	0.001	1.000	0.070	1.000	-0.017	0.070	-0.001
-2.00	2.0	86.0	0.999	0.005	1.001	0.139	1.001	-0.035	0.139	-0.005
-3.00	3.0	84.0	0.997	0.011	1.003	0.209	1.001	-0.052	0.208	-0.011
-4.00	4.0	82.0	0.995	0.020	1.005	0.278	1.002	-0.070	0.277	-0.019
-5.00	5.1	79.9	0.992	0.031	1.008	0.346	1.004	-0.088	0.345	-0.030
-6.00	6.1	77.9	0.989	0.044	1.011	0.413	1.006	-0.106	0.411	-0.043
-7.00	7.2	75.8	0.984	0.060	1.015	0.480	1.008	-0.124	0.477	-0.059
-8.00	8.3	73.7	0.979	0.079	1.020	0.546	1.010	-0.142	0.540	-0.076
-9.00	9.5	71.5	0.973	0.101	1.026	0.610	1.013	-0.160	0.603	-0.095
-10.00	10.7	69.3	0.966	0.125	1.032	0.673	1.016	-0.179	0.663	-0.117
-11.00	12.0	67.0	0.957	0.152	1.039	0.734	1.020	-0.198	0.721	-0.140
-12.00	13.3	64.7	0.947	0.183	1.048	0.794	1.025	-0.218	0.777	-0.165
-13.00	14.7	62.3	0.935	0.216	1.057	0.852	1.030	-0.238	0.830	-0.192
-14.00	16.3	59.7	0.922	0.254	1.067	0.907	1.035	-0.258	0.880	-0.219
-15.00	18.0	57.0	0.905	0.296	1.079	0.960	1.042	-0.279	0.927	-0.248
-16.00	19.9	54.1	0.884	0.344	1.091	1.010	1.049	-0.301	0.971	-0.278
-16.70	21.4	51.9	0.867	0.381	1.102	1.043	1.055	-0.317	0.999	-0.300
-18.00	25.0	47.0	0.822	0.465	1.123	1.101	1.068	-0.347	1.047	-0.340
-19.00	29.3	41.7	0.760	0.558	1.143	1.139	1.081	-0.372	1.077	-0.371
-19.10	30.0	40.9	0.751	0.571	1.145	1.143	1.082	-0.375	1.080	-0.374
-19.20	30.7	40.1	0.739	0.585	1.148	1.146	1.084	-0.377	1.082	-0.377
-19.30	31.6	39.1	0.725	0.603	1.150	1.149	1.086	-0.380	1.085	-0.380
-19.40	32.9	37.7	0.705	0.626	1.153	1.153	1.087	-0.383	1.087	-0.383
-19.47	35.0	35.6	0.672	0.661	1.155	1.155	1.089	-0.385	1.089	-0.385

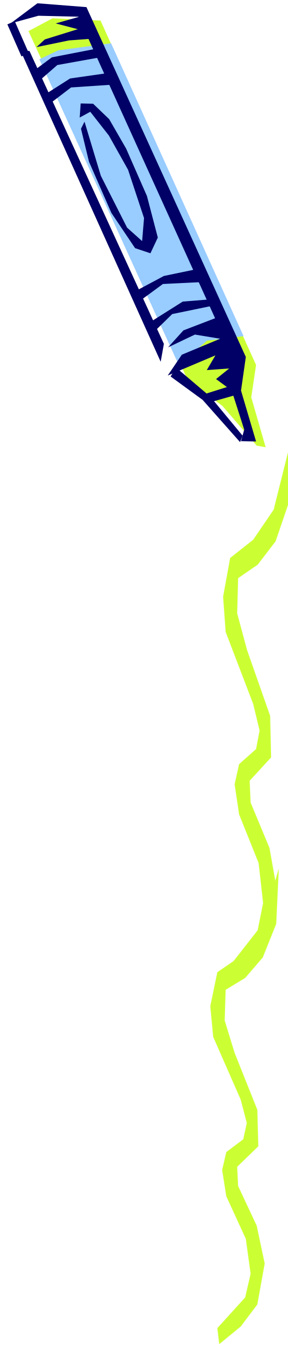
Kelvin波の波頂線（等位相線）

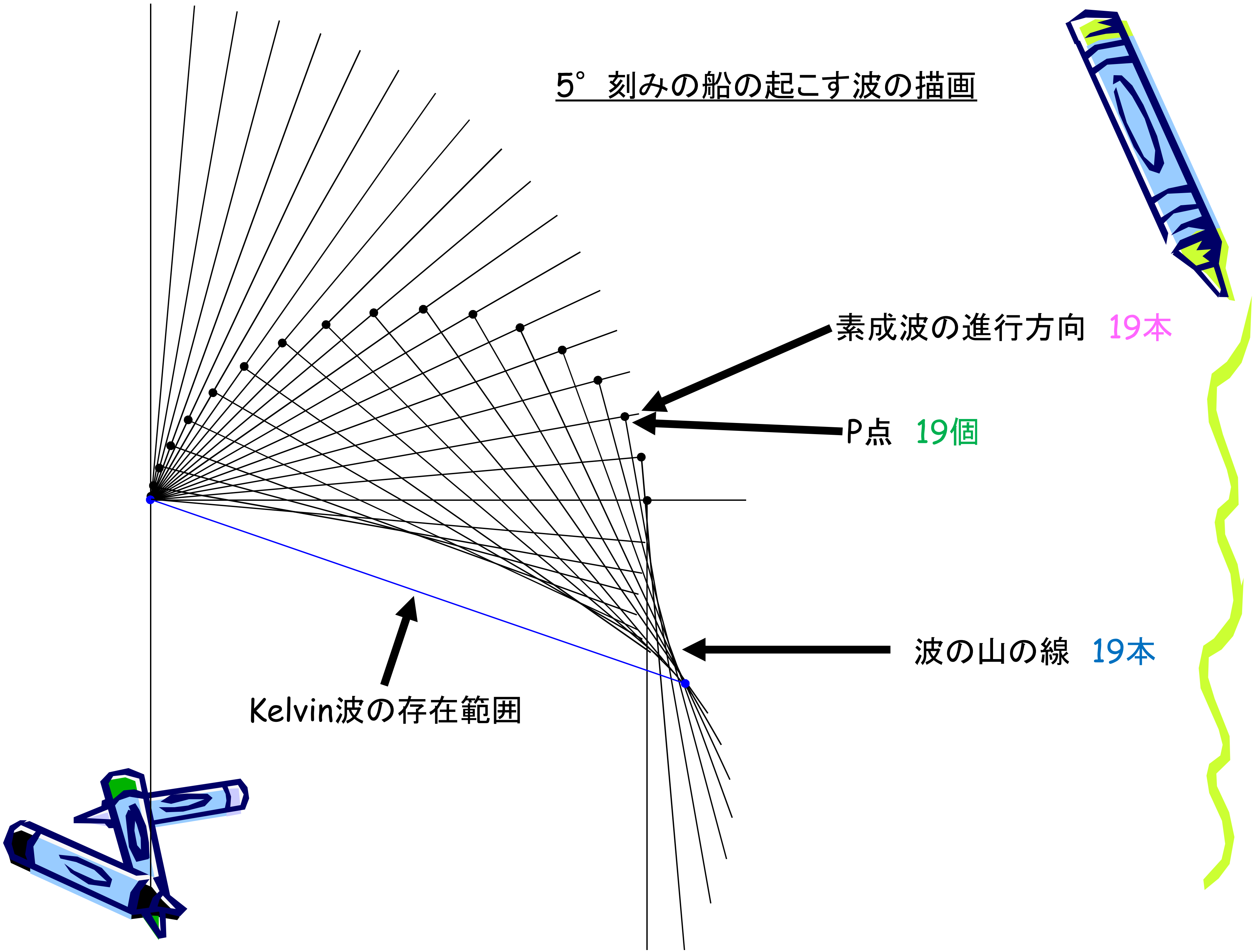


船の波（ケルビン波）を作図してみよう！

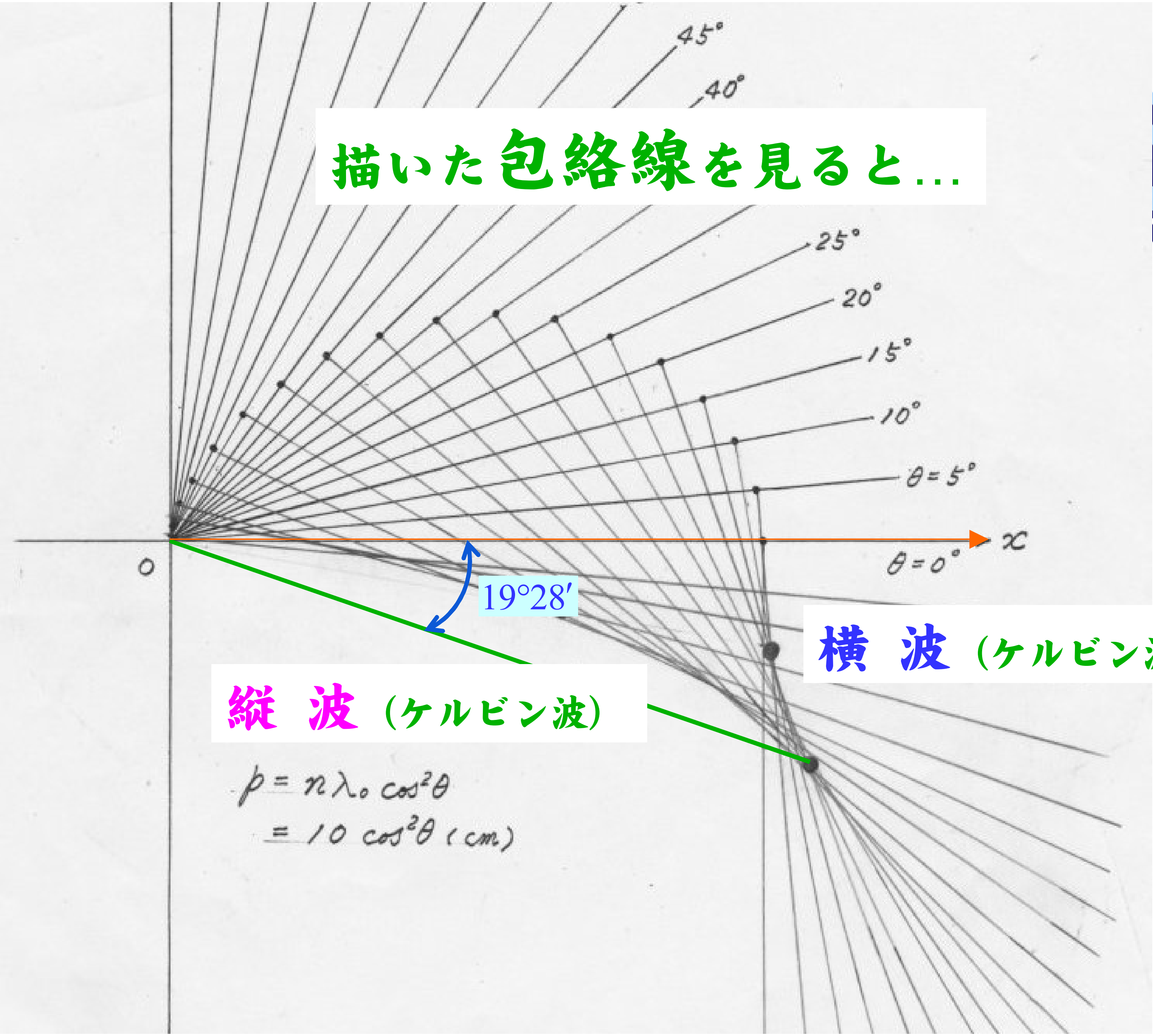


$\theta°$	$\cos\theta$	$10\cos^2\theta$ (cm)
5	0.99619	9.92
10	0.98481	9.70
15	0.96593	9.33
20	0.93969	8.83
25	0.90631	8.21
30	0.86603	7.50
35	0.81915	6.71
40	0.76604	5.87
45	0.70711	5.00
50	0.64279	4.13
55	0.57358	3.29
60	0.50000	2.50
65	0.42262	1.79
70	0.34202	1.17
75	0.25882	0.67
80	0.17365	0.30
85	0.08716	0.08



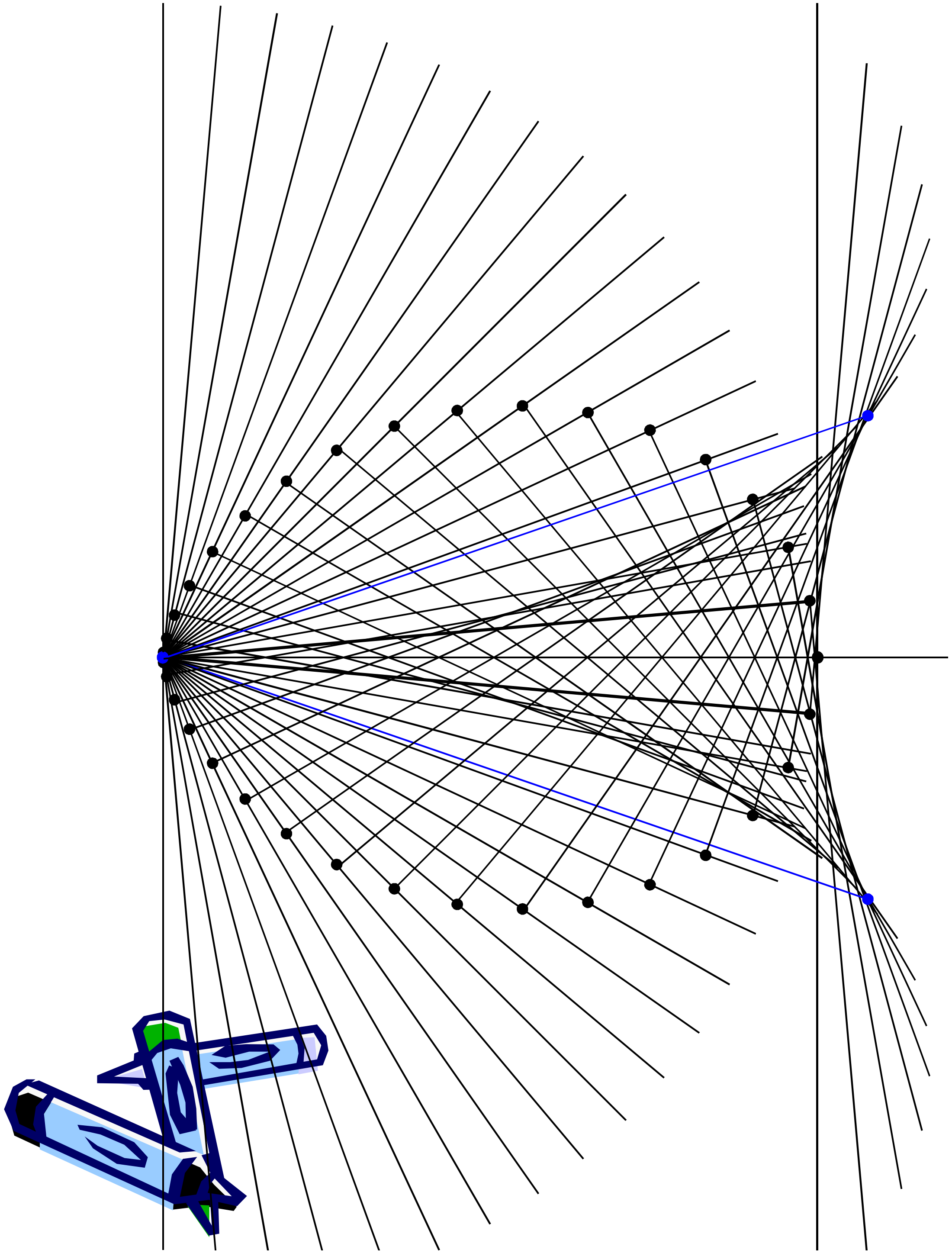

5°刻みの船の起こす波の描画
素成波の進行方向　19本
P点　19個
波の山の線　19本
Kelvin波の存在範囲

描いた包絡線を見ると…

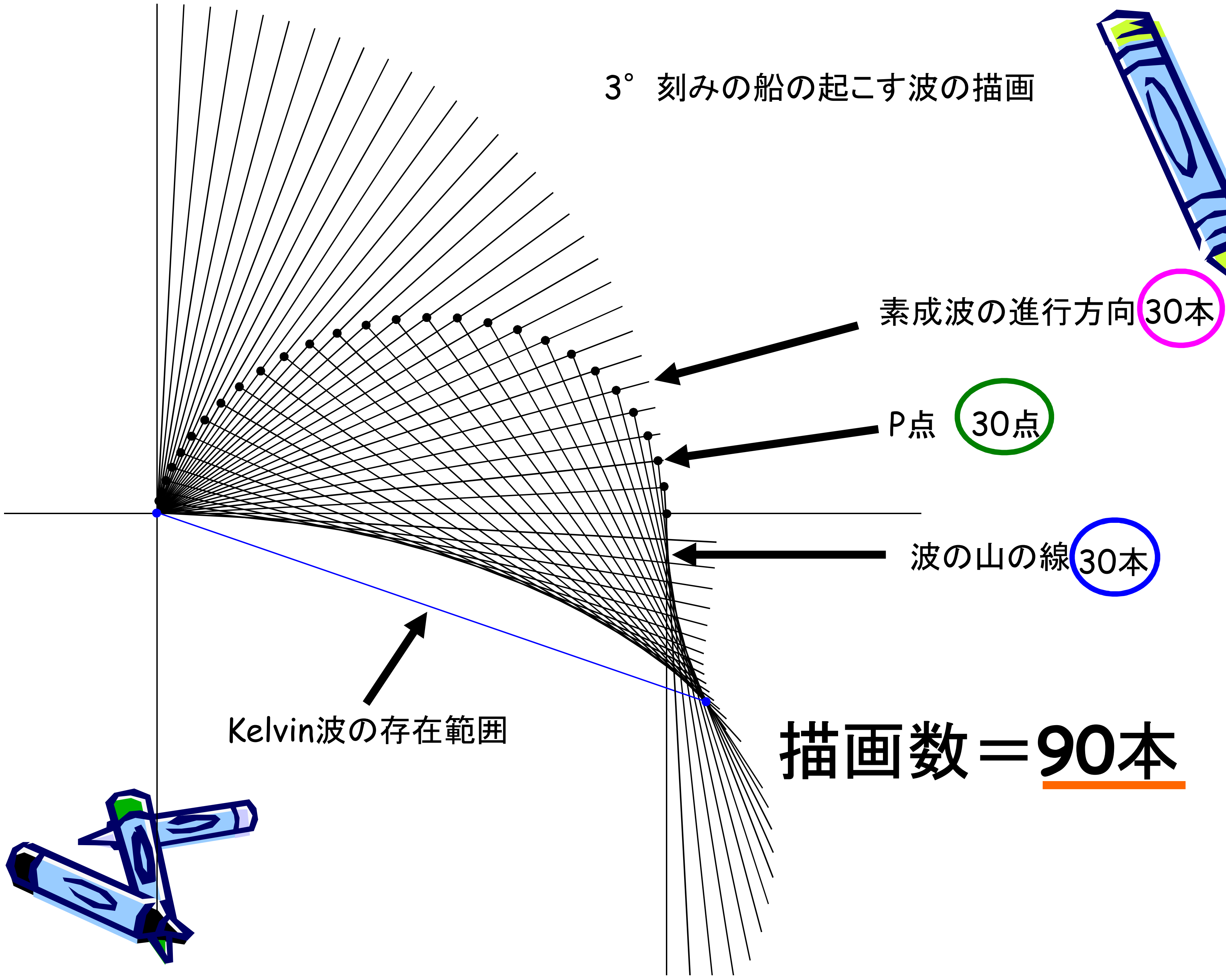


45°
40°
25°
20°
15°
10°
θ=5°
θ=0°
x
O
19°28′
横 波（ケルビン波）
縦 波（ケルビン波）
p = nλ₀ cos²θ
= 10 cos²θ (cm)

5° 刻みの両舷の波の描画

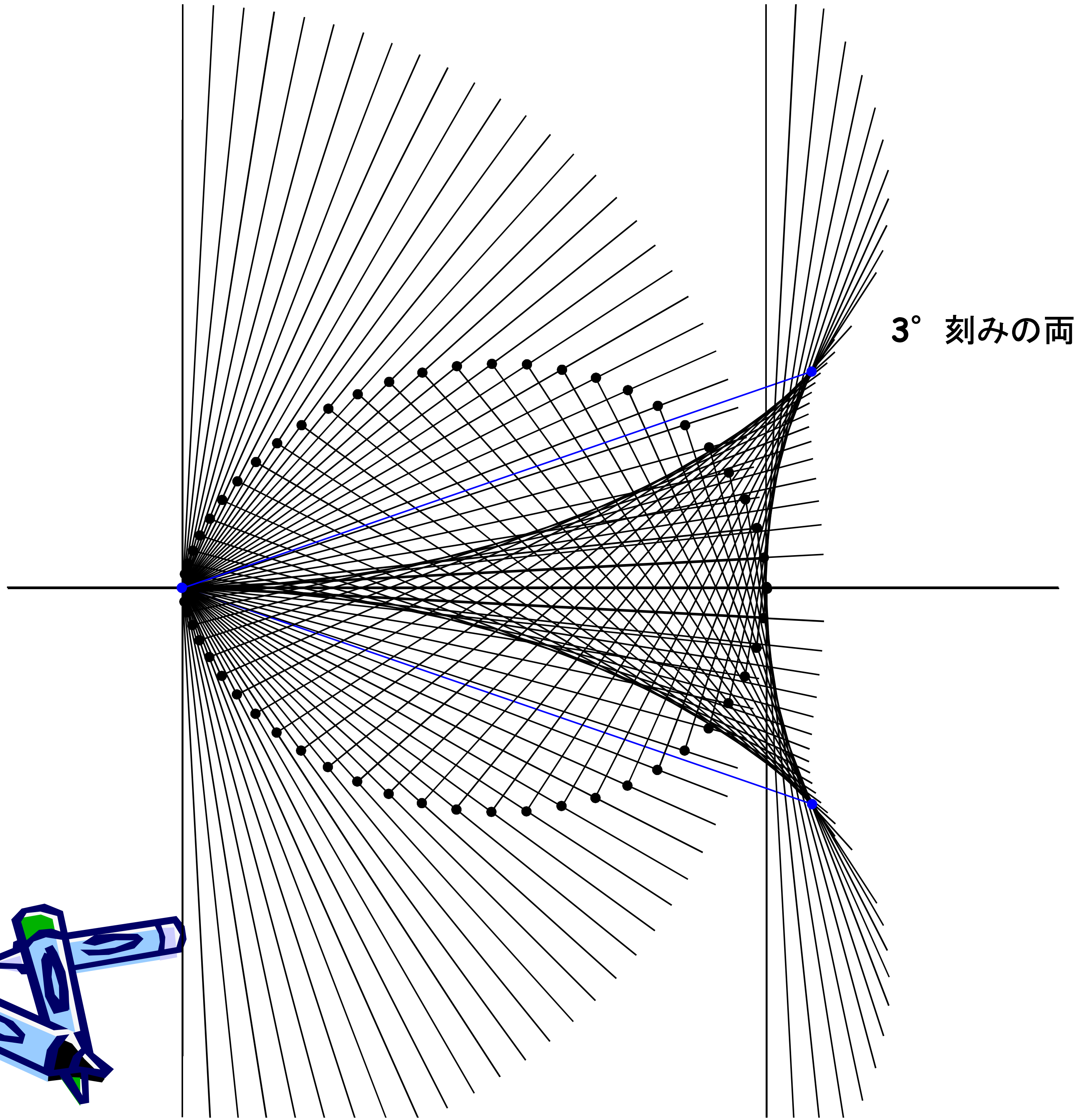


3° 刻みの船の起こす波の描画

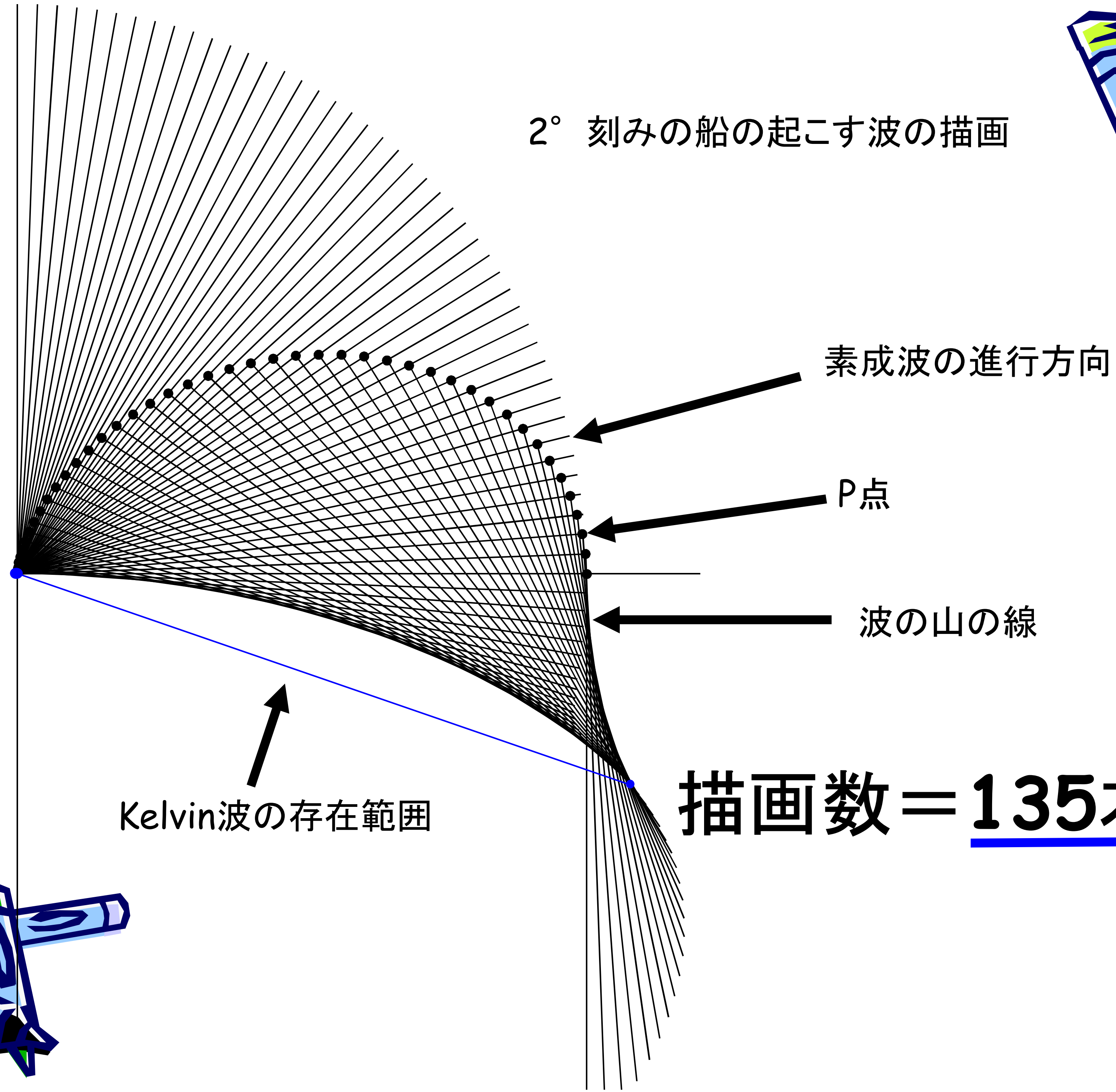


描画数＝90本

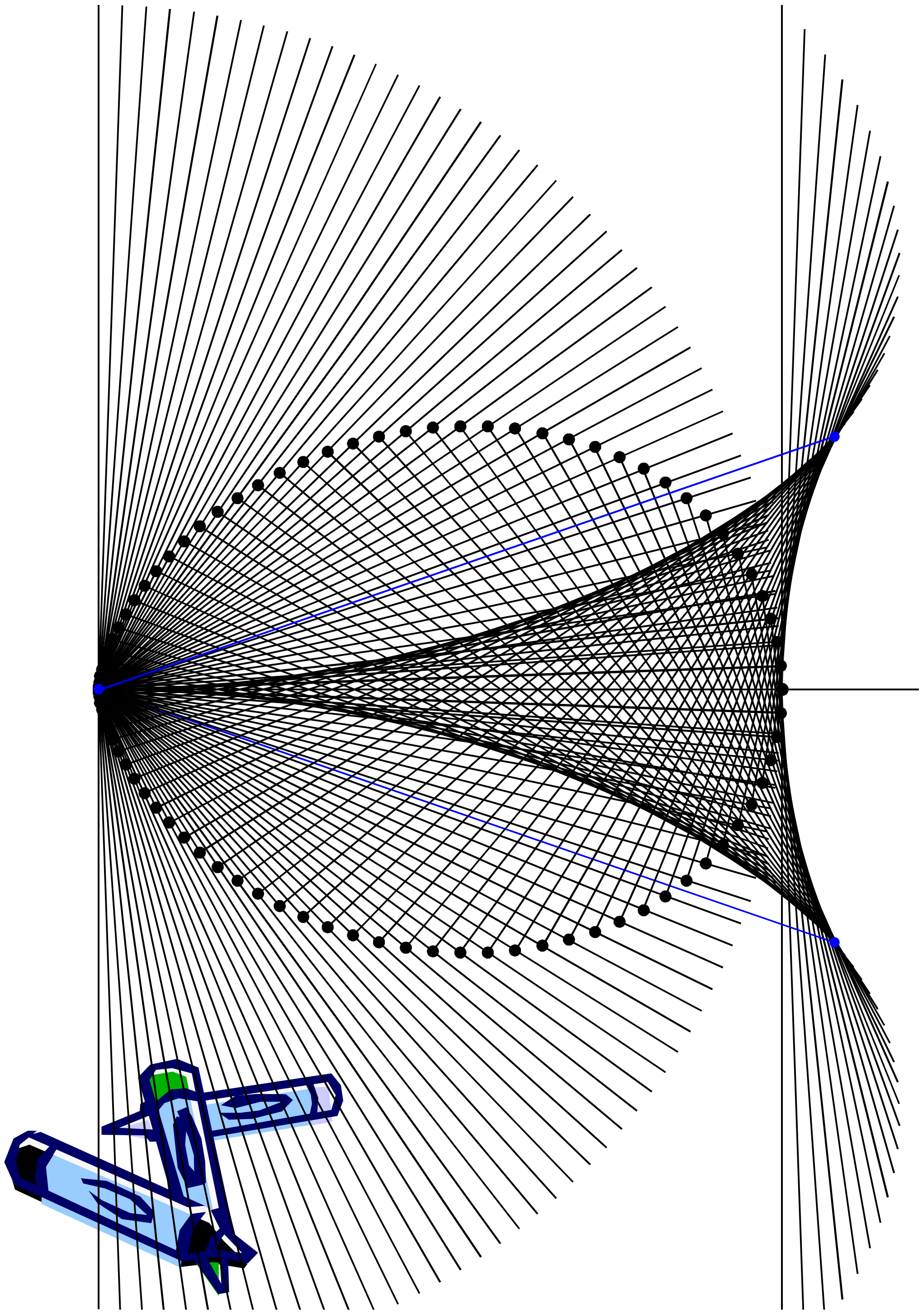
3° 刻みの両舷の波の描画



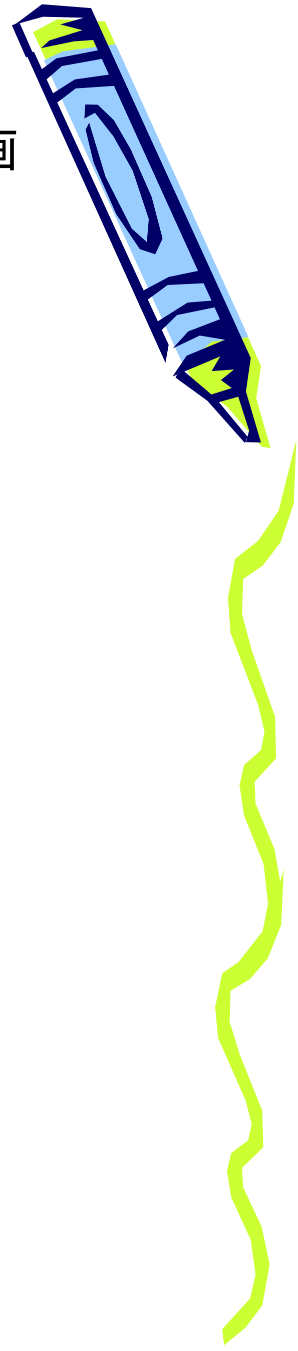
2° 刻みの船の起こす波の描画



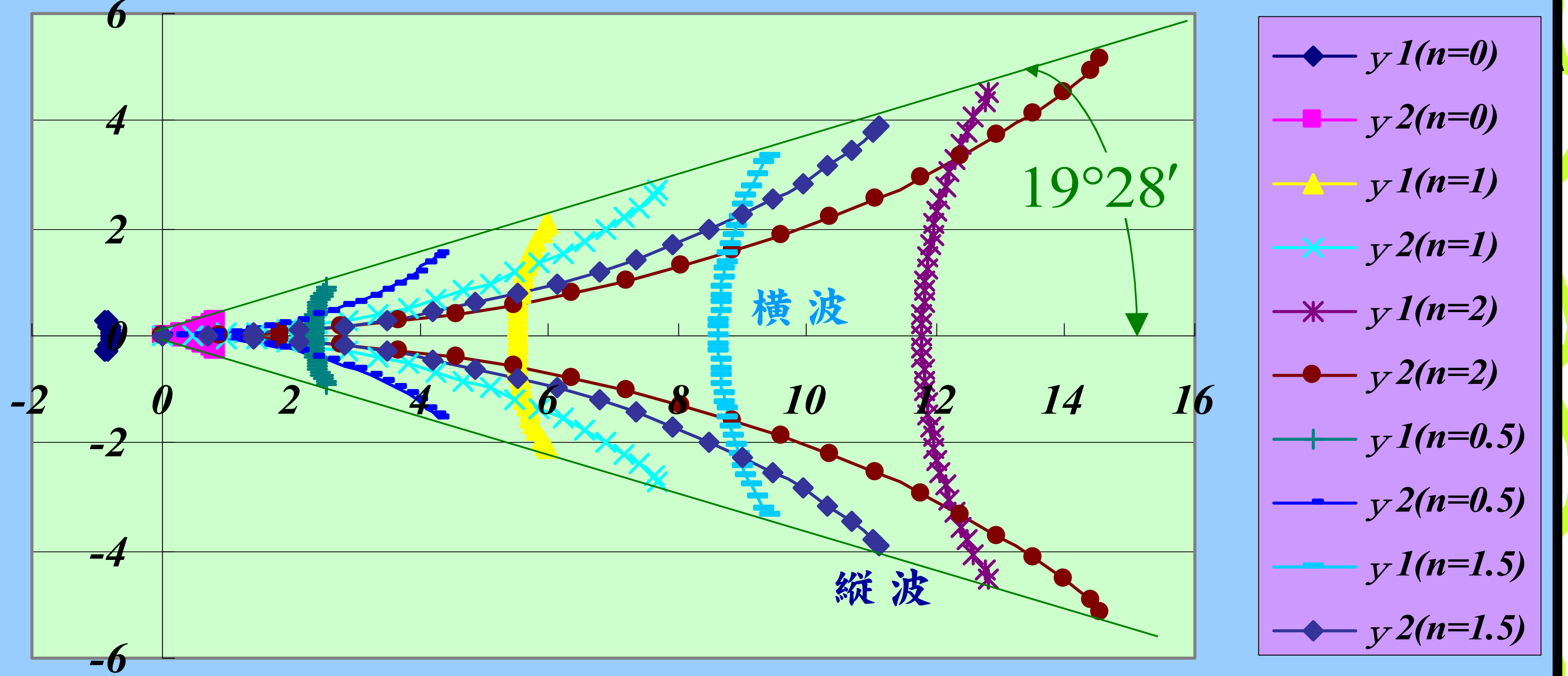
描画数＝135本



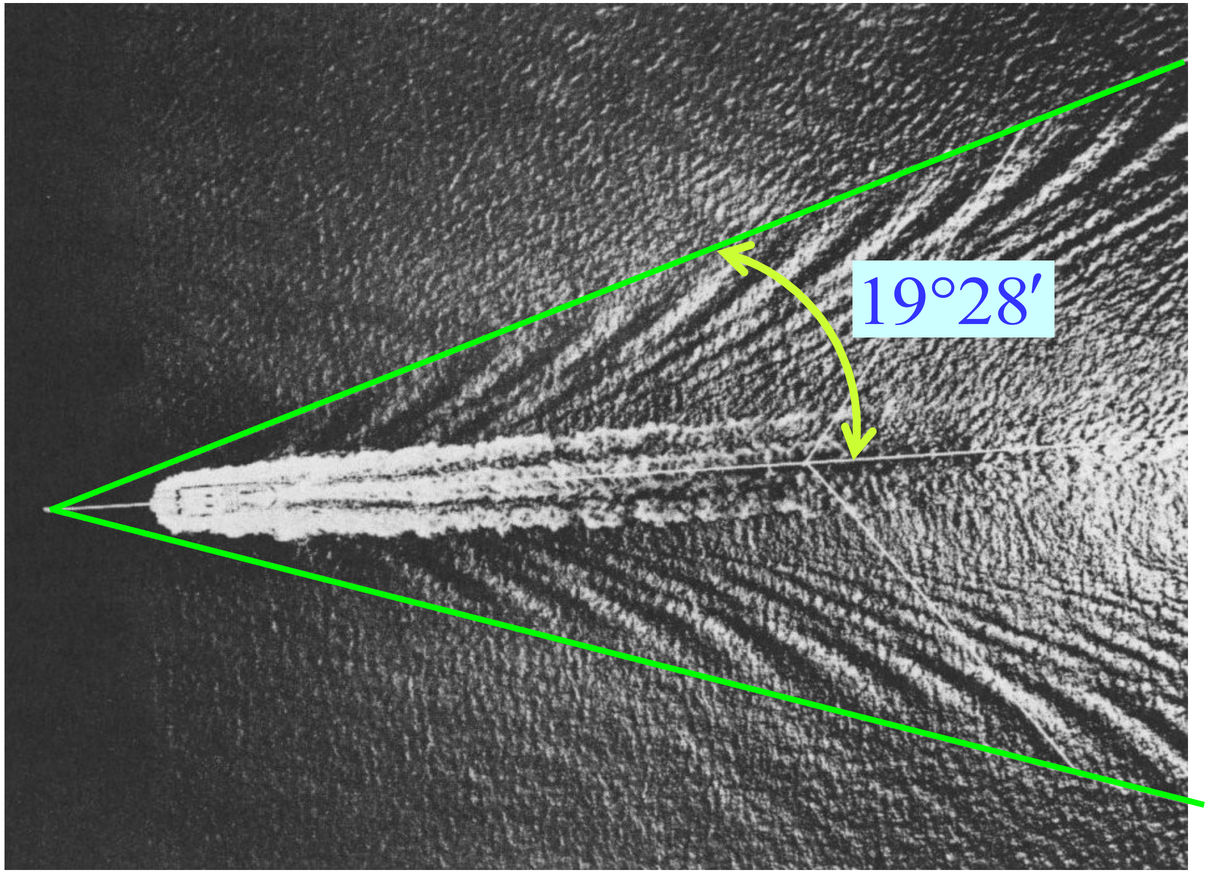
2° 刻みの両舷の波の描画



縦波と横波の波頂線



6
4
2
0
-2
-4
-6
-2
0
2
4
6
8
10
12
14
16
19°28′
横 波
縦 波
y 1(n=0)
y 2(n=0)
y 1(n=1)
y 2(n=1)
y 1(n=2)
y 2(n=2)
y 1(n=0.5)
y 2(n=0.5)
y 1(n=1.5)
y 2(n=1.5)



船の波（ケルビン波）のこと，

少しは(^o^)分かってもらえましたか？...